Algebraic combinatorics final project problems

July 20, 2023

1 Smallest nonabelian finite simple group

Prove that the smallest nonabelian finite simple group is of order 60, and it is isomorphic to A_5.

2 Transposition generators of S_n

Given an arbitrary subset of the set of all transpositions of S_n, count the number of minimal sets of generators of S_n contained in the subset. Namely, construct your best algorithm that computes this number, given any subset of the set of transpositions as an input.

Terminology: by a minimal set of generators of S_n, we mean a set of generators of S_n, such that if any element gets deleted from the set, the remaining elements do not generate S_n.

3 Characters of representations of S_n

Let $\rho : S_n \to GL(V)$ be a finite dimensional representation of S_n over the complex numbers. Are the characters always integer valued? Can you prove your assertions?

4 Graph Laplacian and quadratic forms.

Consider the following pair of graphs Let L_1, L_2 denote the Laplacian of the first and the second graphs,

G_1 $\qquad\qquad$ G_2
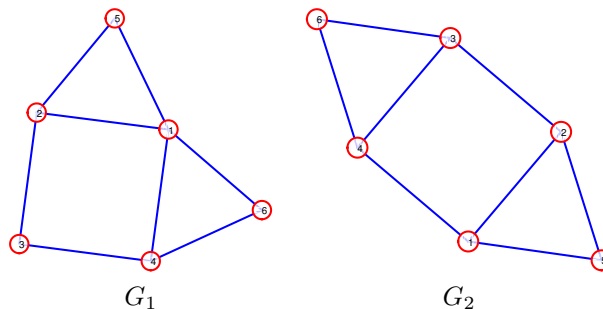
respectively.

1. Show that the equivalence class of the integral quadratic form defined by the Laplacian, depends only on the isomorphism class of the graph.

2. Show that L_1 and L_2 are equivalent as integral quadratic forms.

3. Can you make a conjecture regarding when two graphs have equivalent integral quadratic forms? And conversely, if two graphs have equivalent integral quadratic forms, how are the two graphs related to each other?

4. Try to prove your conjectures.