TSINGHUA MATHCAP 2021 COURSE: NUMBER THEORY

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The distribution of primes and Diophantine equations are main topics in Number Theory. There are some examples: (i) There exist infinitely many primes. (ii) There does not exist right triangle with integral side lengths whose area is a square.

The main goal of this course is to prove the following two results:

(1) Dirichlet's theorem: Given $a, N \geq 1$ coprime integers, there exist infinitely many primes congruent to a modulo N.

(2) Mordell's theorem: For a projective smooth curve
$$E : y^2 z = x^3 + axz^2 + bz^3, \qquad a, b \in \mathbb{Q},$$
the set of rational points on E has a natural finitely generated abelian group structure.

In this course, we will cover some basics on group theory: abelian groups, characters, and structure of finitely generated abelian group. Prerequisite for this course is *Calculus* and *Linear Algebra*.

Homework will be assigned regularly, and some project problems will be proposed.

Group: Groups, Rings, Fields, Finite Abelian groups,
         Characters of groups

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