Diophantine Approximation (course description)

We will study how real numbers can be approximated by rational numbers, more generally, how to approximate complex objects (real numbers) by simple object (rational numbers).

During the first week we will give several introductory lectures in different topics and then each of these topics will be studied in more details.

We will deal with the theory of continued fractions, that is representation of numbers in a form

$$[b_0; b_1, \dots, b_s, \dots] = b_0 + \cfrac{1}{b_1 + \cfrac{1}{b_2 + \cfrac{1}{b_3 + \cdots + \cfrac{1}{b_s + \dots}}}},$$

Farey fractions and Stern-Brocot sequences, related to the procedure of constructing all rational numbers in $[0, 1]$ by means of the operation

$$\frac{a}{b}, \frac{c}{d} \quad \mapsto \quad \frac{a+c}{b+d},$$

Minkowski question mark function $?(x)$, nonlinear approximation and certain related problems. We will discuss how geometric objects (lattices) may be useful for arithmetical problems (for example representation of integers as sums of two and four squares) and deal with some applications to related problems, such as chromatic numbers of certain graphs.

Some basic Number Theory (properties of Euler function $\varphi(n)$ an Möbius function $\mu(n)$, quadratic residues and non-residues, etc...) and Classical Analysis will be also involved.