Homework 3/ **Algebraic combinatorics I**

(1) Let G be a group, and H a subgroup of G. In order for our natural group operation on G/H to be well-defined, is it necessary that H is a normal subgroup of G?

(2) Let $f(x_1, x_2) = a_1 x_1^2 + a_0 x_1 x_2 + a_2 x_2^2$, where the coefficients a_0, a_1, a_2 are elements in the finite field $\mathbb{F}_p$. Give an explicit formula for the number of solutions to $f(x_1, x_2) = 0$ over $\mathbb{F}_b$, and prove your assertion.

(3) Let G be a finite connected graph, and let L denote its combinatorial Laplacian operator. Prove that $Ker(f)$ is 1-dimensional.