LINEAR ALGEBRA HOMEWORK 1

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Exercise 0.1. *WRITE UP by July 18, 2018 Recall that for $a, b \in \mathbb{Z}$, $a \neq 0$, the fraction b/a is defined to be the set of equations*

$$nx = m, \quad n, m \in \mathbb{Z}, \quad n \neq 0, \ s.t. \ nb = ma.$$

Prove that b/a is not empty. Prove that two such sets b/a and b'/a' are either disjoint or equal.

Exercise 0.2. *Verify that the multiplication rule given by*

$$\times : \mathbb{Q}^2 \to \mathbb{Q}, \ (b/a, b'/a') \mapsto (bb')/(aa')$$

is well-defined. Note that this rule is generalizes the usual rule of multiply integers, i.e. grouping apples.

Exercise 0.3. *Consider the map $\iota : \mathbb{Z} \to \mathbb{Q}$, $n \mapsto n/1$. Is ι injective, surjective? Prove your assertions. Verify the identities*

$$\iota(1) = 1/1, \quad \iota(nn') = \iota(n)\iota(n'), \quad n, n' \in \mathbb{Z}.$$

Also express $\iota(n + n')$ in terms of $\iota(n)$, $\iota(n')$.

Exercise 0.4. *Assume we have a field $\mathbb{C}$ with properties ($\mathbb{C}1$)-($\mathbb{C}3$). Use N1-N9 to verify that for $z, w \in \mathbb{C}$, $z + w = 0$ iff $w = -z$. Now show that for $x, y \in \mathbb{R}$*

$$-(x + iy) = -x + i(-y).$$

In fact, in any field F, N1-N9 imply that $-(a+b) = (-a)+(-b)$ and $-(ab) = (-a)b$.

Exercise 0.5. *WRITE UP by July 20, 2018 If $u = (u_1, u_2), u' = (u'_1, u'_2) \in \mathbb{R}^2$ are two solutions to $x_1 + x_2 = 1$, what can we say about $u - u'$? Find a nice (as nice as you can think of) bijection between solution set of this equation and the solution set of its homogeneous "partner" $x_1 + x_2 = 0$. Can you interpret your bijection geometrically? How would you generalize your results?*

Exercise 0.6. *Let F be a field. For $X, Y, Z \in F^2$ and $\lambda, \mu \in F$, verify that*
V1. $(X + Y) + Z = X + (Y + Z)$
V2. $X + Y = Y + X$
V3. $X + 0 = X$
V4. $X + (-X) = 0$
V5. $\lambda(X + Y) = \lambda X + \lambda Y$
V6. $(\lambda + \mu)X = \lambda X + \mu X$
V7. $(\lambda\mu)X = \lambda(\mu X)$
V8. $1X = X$.
The same holds true for F^n.
Suggestion: *Think about exactly what facts about F you need to use to prove each of these statements.*