Probability and Statistics
Tsinghua Math Camp Prof. Paul Horn
Homework 3: Due Wednesday 7/26/2016

Note: You should get to know your coaches, Gavin St. John (`gavin.stjohn@du.edu`) and Zhe Liu (刘喆) (`z-liu16@mails.tsinghua.edu.cn`). If you need to contact me, my email address is `paul.horn@du.edu` or you may ask via WeChat.

I want very much to learn all of your names, but please be patient with me as I am not so good at pronouncing Chinese and I am very bad at remembering names (even non-Chinese names). So please help me by telling me your name very slowly and being patient with me as I try to learn and pronounce it.

Homework problems: To be turned in.

1. Suppose X is a Geometric(p) random variable, so that $f_X(k) = (1-p)^k - 1 \cdot p$. Find $\mathbb{E}[X]$ and $\text{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$. (Note/hint, relate $\mathbb{E}[X]$ to a derivative of a geometric sum.)

2. Roll a die, look at the resulting value, and flip that many coins. Let X be the number of heads obtained. Find $f_X(x)$ and $\mathbb{E}[X]$. (Note: the number of coins flipped is random.)

3. We flip a coin until we get two *consecutive* heads. Let X denote the number of coin flips performed. Find $f_X(k)$.

4. Suppose X and Y are independent Poisson random variables. That is,
$$\mathbb{P}(X = k) = e^{-\lambda_1}\frac{\lambda_1^k}{k!} \qquad \mathbb{P}(Y = k) = e^{-\lambda_2}\frac{\lambda_2^k}{k!},$$
for $0 \le k \le \infty$ and also that $\mathbb{P}(X = k, Y = j) = \mathbb{P}(X = k)\mathbb{P}(Y = j)$. (To be clear, the first of these is being Poisson, while the second of these is the independence.) What is the distribution of $X + Y$? (Be sure to simplify to get a nice answer!)

5. Let $U_1, \ldots, U_n$ be independent, and uniformly random numbers from $1, \ldots, k$. Let $X = \max\{U_1, \ldots, U_n\}$. Find $f_X(k)$.

6. Suppose X is normally distributed with parameters μ and σ. Let $Y = e^X$. Find the pdf of Y.