

# TSINGHUA MATHCAMP 2017 COURSE: NUMBER THEORY COURSE DESCRIPTION

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Number Theory is one of the oldest and most extensively studied branches of mathematics. At its core number theory studies properties of the integers and their arithmetic, including questions about primes and divisibility, and solutions to polynomial equations. For example, one famous problem we'll study is to determine which prime numbers can be written in the form  $x^2 + y^2$  where  $x$  and  $y$  are integers.

There are many number theory questions which are easy to state, and which involve familiar objects, but whose solutions turn out to involve beautiful and interesting ideas. In that spirit, our motto for the course will be to "Think Deeply of Simple Things." We'll approach the material in a spirit of exploration, with students being encouraged to look for patterns, make conjectures, and develop proofs. Our goal will be to learn to think like mathematicians as we delve into the subject and uncover its hidden structure.

Topics to be covered include congruences, continued fractions, unique factorization, the geometry of numbers, and extensions to systems beyond the integers.

The only background required for the course is high school algebra, and an eagerness to explore. Homework will be assigned regularly during the course, and there will be a research project that expands on the ideas we will be discussing. Mathcampers will meet daily with our coach to work on the homework and project, and are very much encouraged to work on the material with each other as well.