Analysis & Topology

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In this course, we focus on the theory of the decomposition and superposition of waveforms (one dimensional functions of time), **Fourier Series** and the **Fourier Transform**, which give a fundamental understanding of waves — the most common and important physical existence and concept.

We start with the exponential function e^x, the most important function in mathematics according to the famous mathematician in analysis Rudin, focusing on its special property in the "eigen" sense for differentiation. We study its important role in solving simple ordinary differential equations for the physics of oscillation, where we establish the complex exponential from Euler's formula. Using these we demonstrate the principle of **resonance** in an input-response model. Then we introduce the Fourier series and explore various waveforms that can be represented by their Fourier series. At last, we generalize the Fourier coefficients to the Fourier Transform (FT) of underlying functions and consider some important applications of FT.

The whole theory, later evolving into the branch of *harmonic analysis,* is to explore how the time-dependence of a wave is mathematically related to its frequencies. This observation has various historically important implications including the uncertainty principle.